EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 26, 849–858 (1997)

EVALUATION OF BUILDING SEPARATION DISTANCE REQUIRED TO PREVENT POUNDING DURING STRONG EARTHQUAKES

JOSEPH PENZIEN*

International Civil Engineering Consultants, Inc., Berkeley, CA 94704, U.S.A.

SUMMARY

Presented herein is an analytical procedure for evaluating the separation distance required between two buildings to prevent pounding during strong earthquakes. The procedure is based on equivalent linearization of non-linear hysteretic behaviour and application of the well-known CQC method of weighting normal mode responses. The numerical results obtained are compared with corresponding results obtained using the well-known SRSS and ABS methods. © 1997 by John Wiley & Sons, Ltd.

Earthquake Engng. Struct. Dyn., **26**, 849–858 (1997).

No. of Figures: 8. No. of Tables: 0. No. of References: 5.

KEY WORDS: pounding of buildings

INTRODUCTION

Major damage to buildings in metropolitan areas is often caused by the pounding action of one against another during strong motion earthquakes. To avoid such damage, new buildings located adjacent to each other should be provided with adequate separation distances. To specify minimum distances for this purpose, a rational analysis procedure is needed. It is this need that stimulated the author to develop the procedure presented herein. Hopefully, it can provide a basis for developing a code provision specifying minimum separation distances.

LINEAR RESPONSE

Consider two buildings separated by a distance Δ and having heights H_1 and H_2 as shown in Figure 1. If both buildings are subjected to the same ground acceleration $\ddot{v}_{gy}(t)$ in the y direction, the normal mode equations of motion for the ith and jth modes of Buildings 1 and 2 will be

$$\ddot{Y}_{1i}(t) + 2\omega_{1i}\xi_{1i}\dot{Y}_{1i}(t) + \omega_{1i}^2 Y_{1i}(t) = -\frac{L_{1i}}{M_{1i}}\ddot{v}_{gy}(t) \tag{1}$$

$$\ddot{Y}_{2j}(t) + 2\omega_{2j}\xi_{2j}\dot{Y}_{2j}(t) + \omega_{2j}^2 Y_{2j}(t) = -\frac{L_{2j}}{M_{2j}}\ddot{v}_{gy}(t) \tag{2}$$

respectively, in which

$$L_{1i} \equiv \int_0^{H_1} m_1(x)\phi_{1i}(x)\,\mathrm{d}x, \qquad M_{1i} \equiv \int_0^{H_1} m_1(x)\phi_{1i}(x)^2\,\mathrm{d}x, \quad i = 1, 2, \ldots, N_1 \tag{3}$$

* Correspondence to: J. Penzien, International Civil Engineering Consultants, lnc., 1995 University Avenue, Suite 119, Berkeley, CA 94704, U.S.A.

CCC 0098–8847/97/080849–10$17·50

Received 22 August 1996
Revised 27 February 1997

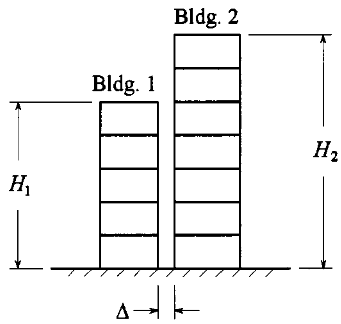

Figure 1. Two buildings of heights H_1 and H_2 having a separation distance Δ: $H_1 \leqslant H_2$

$$L_{2j} \equiv \int_0^{H_2} m_2(x)\phi_{2j}(x)\,\mathrm{d}x, \qquad M_{2j} \equiv \int_0^{H_2} m_2(x)\phi_{2j}(x)^2\,\mathrm{d}x, \quad j = 1, 2, \ldots, N_2 \tag{4}$$

where N_1 and N_2 denote the number of lower modes to be considered for Buildings 1 and 2, respectively; see Figure 4(a)

In formulating a procedure for setting the separation distance Δ required to avoid seismic pounding, the inward relative displacement $v_R(t)$ between the two buildings at the top level of the shorter building (Bldg. 1) as expressed by

$$v_R(t) = \sum_{i=1}^{N_1} \phi_{1i}(H_1)Y_{1i}(t) - \sum_{j=1}^{N_2} \phi_{2j}(H_1)Y_{2j}(t) \tag{5}$$

is the response quantity of interest. When $v_R(t)$ reaches the value Δ, pounding will occur at level $x = H_1$.

Equation (5) can be expressed in the equivalent form

$$v_R(t) = \sum_{m=1}^{(N_1+N_2)} A_m Y_m(t) \tag{6}$$

in which

$$A_m \equiv \phi_{1m}(H_1), \qquad Y_m(t) \equiv Y_{1m}(t), \quad m = 1, 2, \ldots, N_1 \tag{7}$$

$$A_m \equiv -\phi_{2,m-N_1}(H_1), \qquad Y_m(t) \equiv Y_{2,m-N_1}(t), \quad m = (N_1+1), (N_1+2), \ldots, (N_1+N_2) \tag{8}$$

Using the CQC method,[1,2] the expected maximum absolute value of $v_R(t)$ can be expressed in the form

$$|v_R(t)|_{\max} = \left[\sum_{m=1}^{(N_1+N_2)} \sum_{n=1}^{(N_1+N_2)} A_m A_n \frac{L_m L_n}{M_m M_n} \rho_{mn} S_d(\xi_m, \omega_m) S_d(\xi_n, \omega_n)\right]^{1/2} \tag{9}$$

where

$$L_m \equiv L_{1m}, \qquad M_m \equiv M_{1m}, \quad m = 1, 2, \ldots, N_1 \tag{10}$$

$$L_m \equiv L_{2,m-N_1}, \qquad M_m \equiv M_{2,m-N_1}, \quad m = (N_1+1), (N_1+2), \ldots, (N_1+N_2) \tag{11}$$

and

$$\rho_{mn} = \rho_{nm} = \frac{8\sqrt{\xi_m \xi_n}(\xi_m + r\xi_n)r^{3/2}}{(1-r^2)^2 + 4\xi_m\xi_n r(1+r^2) + 4(\xi_m^2 + \xi_n^2)r^2} \tag{12}$$

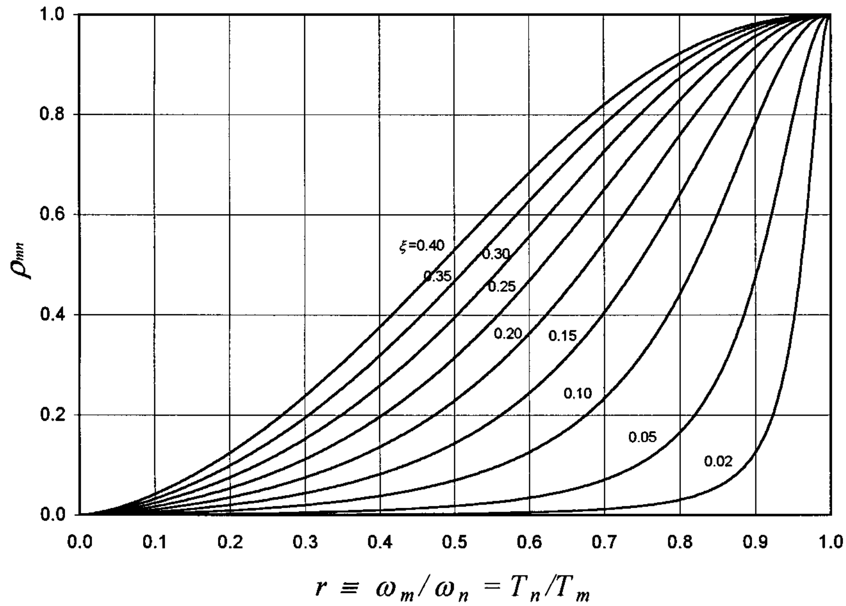

Figure 2. Plots of ρ_{mn} versus r for discrete values of ξ as given by equation (16)

in which

$$r \equiv \omega_m/\omega_n, \qquad \omega_n \geqslant \omega_m \tag{13}$$

$$\omega_m \equiv \omega_{1m}, \qquad \xi_m \equiv \xi_{1m}, \quad m = 1, 2, \ldots, N_1 \tag{14}$$

$$\omega_m \equiv \omega_{2,m-N_1}, \qquad \xi_m = \xi_{2,m-N_1}, \quad m = (N_1+1), (N_1+2), \ldots, (N_1+N_2) \tag{15}$$

The quantity $S_d(\xi_m, \omega_m)$ in equation (9) representing the earthquake ground acceleration $\ddot{v}_{gy}(t)$ is the elastic displacement response spectral value for damping ratio ξ_m and frequency ω_m. If all damping ratios ξ_m $(m = 1, 2, \ldots, N_1 + N_2)$ equal the same value ξ, equation (12) reduces to the simplified form

$$\rho_{mn} = \rho_{nm} = \frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} \tag{16}$$

which is plotted in Figure 2 as a function of r for discrete values of ξ.

Since the value of $|v_R(t)|_{max}$ is usually dominated by the first-mode responses of Buildings 1 and 2, the higher-mode responses can be ignored, in which case equation (6) simplifies to

$$v_R(t) = \phi_{11}(H_1)Y_{11}(t) - \phi_{21}(H_1)Y_{21}(t) = \sum_{s=1}^{2} B_s Y_s(t) \tag{17}$$

in which

$$\begin{aligned} B_1 &\equiv \phi_{11}(H_1), \qquad B_2 \equiv -\phi_{21}(H_1) \\ Y_1(t) &\equiv Y_{11}(t), \qquad Y_2(t) \equiv Y_{21}(t) \end{aligned} \tag{18}$$

The CQC expression for $|v_R(t)|_{max}$, equation (9), then reduces to

$$|v_R(t)|_{max} = \left[\sum_{s=1}^{2} \sum_{t=1}^{2} B_s B_t \frac{L_s L_t}{M_s M_t} \rho_{st} S_d(\xi_s, \omega_s) S_d(\xi_t, \omega_t) \right]^{1/2} \tag{19}$$

If the first-mode frequencies of Buildings 1 and 2 are widely separated and the corresponding damping ratios are small, the cross terms in equation (19) become negligible. In this case $|v_R(t)|_{max}$ can be approximated by the more conservative SRSS expression

$$|v_R(t)|_{max} = \left[\sum_{s=1}^{2} B_s^2 \frac{L_s^2}{M_s^2} S_d(\xi_s, \omega_s)^2 \right]^{1/2} \tag{20}$$

An even more conservative relation is given by the ABS (absolute sum) expression

$$|v_R(t)|_{max} = \sum_{s=1}^{2} B_s \frac{L_s}{M_s} S_d(\xi_s, \omega_s) \tag{21}$$

The above procedure, based on linear response, has been developed in a previous paper[3] using random vibration theory; however, the same results can be obtained by a deterministic approach[2] which has been adopted in the above development.

NON-LINEAR HYSTERETIC RESPONSE

The above procedure for evaluating $|v_R(t)|_{max}$ is based on linear elastic response of both Bldgs. 1 and 2; however, since they normally will respond inelastically during strong earthquakes, one must carefully examine the applicability of this procedure in such cases. Similar to an elastic case, $|v_R(t)|_{max}$ will be controlled primarily by first-mode type of responses, even though inelastic deformations take place. Because of the commonly used weak-girder/strong-column principle of design, one can assume the shape of this type of response to be linear as shown in Figure 3, so that $\phi(x) = x/H$. The corresponding generalized SDOF system representing such response is shown in Figure 4(b). The generalized coordinate $Y_e(t)$ corresponds to the displacement at the top of the building relative to the moving ground. The generalized mass M will be given by the second of equations (3), whether or not the response remains elastic, so long as the linear mode-shape function $\phi(x)$ does not change significantly with respect to time. When responding in the linear elastic range as represented in Figure 4(a), the generalized first-mode spring constant K will equal $\omega^2 M$ and its generalized dashpot coefficient C will equal $2M\omega\xi$, where ξ is the first-mode damping ratio.

When responding in the inelastic range, the equivalent linearized SDOF system, as represented in Figure 4(b), can be used to predict maximum response, provided its generalized spring constant K_e and generalized damping coefficient C_e are selected properly. Assuming a bilinear force-displacement hysteretic relation for the non-linear generalized spring as shown in Figure 5, the equivalent linearized spring constant K_e and damping ratio ξ_e can be obtained using the following procedure[4]:

(1) The values of the equivalent-linearized stiffness K_e and damping ratio ξ_e depend upon the value of Y_u, which is a weighted value of $|Y_e(t)|_{max}$ as expressed by

$$Y_u = \alpha |Y_e(t)|_{max} \tag{22}$$

The value of α which yields the optimum linearized system depends upon the characteristics of the ground-motion input $v_{gy}(t)$; however, for typical seismic inputs, the value 0·65 gives reasonably realistic results.

The value of K_e, as shown by the slope of the dashed line 402 in Figure 5, is given by

$$K_e = K \left[\frac{\gamma + \beta(\mu - \gamma)}{\mu} \right] \tag{23}$$

in which

$$\gamma \equiv 1/\alpha = |Y_e(t)|_{max}/Y_u \tag{24}$$

$$\beta \equiv \bar{K}/K \tag{25}$$

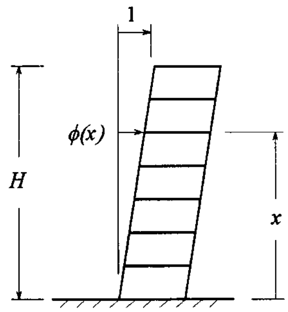

Figure 3. Assumed linear displacement shape for both elastic and inelastic response

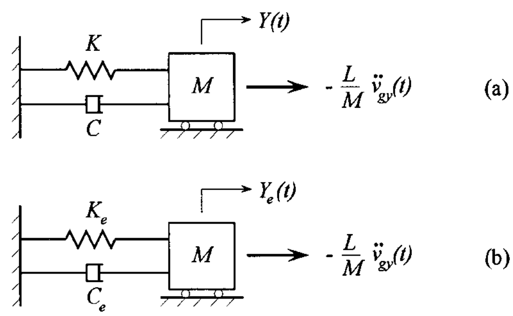

Figure 4. Generalized first-mode SDOF systems representing elastic and inelastic responses of the building in Figure 3

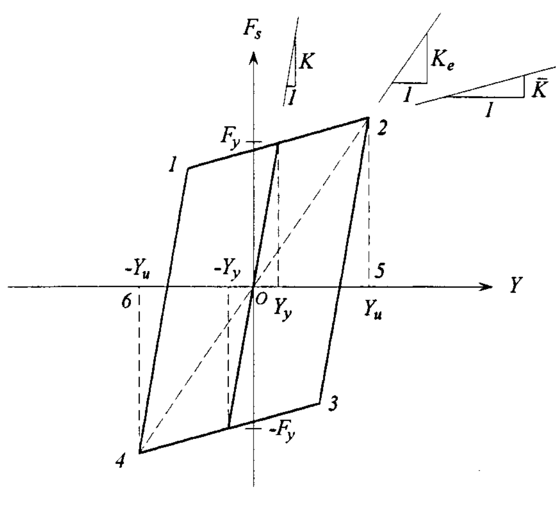

Figure 5. Bilinear force–displacement hysteretic relation for non-linear generalized spring

and μ is the maximum ductility demand defined by

$$\mu \equiv |Y_e(t)|_{max}/Y_y \tag{26}$$

The period T_e of the equivalent-linearized system can now be obtained using

$$T_e = 2\pi\sqrt{M/K_e} \tag{27}$$

(2) The viscous damping ratio ξ_e is evaluated as the sum of that ratio ξ_{e1} considered appropriate for response in the linear elastic range, usually assigned the value 0·05, and that ratio ξ_{e2} which will produce an energy loss E_v during a full cycle of oscillation at frequency $\omega_e = \sqrt{K_e/M}$ and amplitude Y_u that is equal to the hysteretic energy loss E_h during the same full cycle of oscillation. It is easily shown[2] that E_v is given by

$$E_v = 2\pi\xi_{e2} M\omega_e^2 Y_u^2 = 2\pi\xi_{e2} K_e Y_u^2 \tag{28}$$

in which $K_e Y_u^2$ is the sum of the areas under triangles 025 and 046 in Figure 5. The hysteretic energy loss E_h is the area under parallelogram 1234 in this same figure, which can be evaluated using

$$E_h = 4Y_u(Y_u - Y_y)(K - \bar{K}) \tag{29}$$

Setting the right-hand side of equation (28) equal to the right-hand side of equation (29), making use of equations (22)–(26), and then solving for ξ_{e2} yields

$$\xi_{e2} = \frac{2}{\pi}\left\{\frac{(u-\gamma)(1-\beta)\gamma}{\mu[\gamma + \beta(\mu - \gamma)]}\right\} \tag{30}$$

Specifying ξ_{e1} equal to 0·05 and adding this value to ξ_{e2} as given by equation (30) and using $\gamma = 1/\alpha = 1/0{\cdot}65 = 1{\cdot}54$, functions of ξ_e vs. μ can be generated for discrete values of β as shown in Figure 6.

Having established the equivalent-linearized stiffness K_e and damping ratio ξ_e for each building, equation (12) can be used to generate the cross-term ρ_{e12}, which equals ρ_{e21}, by introducing the corresponding equivalent-linearized value of r define by

$$r_e \equiv \frac{T_{e11}}{T_{e21}}, \qquad T_{e21} \geqslant T_{e11} \tag{30}$$

in which

$$T_e = 2\pi\sqrt{M/K_e} \tag{31}$$

The high values of ρ_e, obtained for typical values of ductility demand, result in large values for the cross-terms as compared with the corresponding values obtained for low-damped ($\rho = 0{\cdot}05$) linear elastic response.

For buildings having fundamental periods in the range $0{\cdot}5 < T < 3{\cdot}0$ s, the maximum values of the generalized displacements $Y_{11}(t)$ and $Y_{21}(t)$ for the non-linear hysteretic model, Figure 4(b), will be approximately the same as for the linear elastic models, Figure 4(a); thus, the elastic displacement response spectral values can be used in equation (19) when evaluating $|v_r(t)|_{max}$ for the non-linear models. Often, as in the 1994 Uniform Building Code 5 (Figure 16–3, pp. 2–42), these displacement response spectral values, $S_d(\xi, T)$, are specified to be proportional to T over the above-mentioned period range. In this case, for $\xi_{11} = \xi_{21} = \xi$, the following relation is satisfied:

$$S_d(\xi, T_{21}) = \frac{T_{21}}{T_{11}} S_d(\xi, T_{11}) \tag{32}$$

Assuming a uniform mass distribution over the height of each building, the linear mode shape, $\phi(x) = x/H$, yields 3/2 for all values of L_s/M_s and L_t/M_t in equation (19); and, the values of B_1 and B_2 in this same equation equal 1 and H_1/H_2, respectively. The ratio H_1/H_2 can be expressed in terms of the period ratio T_1/T_2 using available empirical relations. For example, the above-mentioned Uniform Building Code specifies that T for all buildings can be determined using the approximate relation

$$T = C_t H^{3/4} \tag{33}$$

in which the coefficient C_t depends upon type of building (C_t equals 0·035 for steel moment-resisting frames, 0·030 for reinforced concrete moment-resisting frames and eccentrically braced frames, and 0·020 for all other buildings) and where H is measured in units of feet. In this case, the period ratio T_{11}/T_{21} is expressed by

$$(T_{11}/T_{21}) = (C_{t1}/C_{t2})(H_1/H_2)^{3/4} \tag{34}$$

This equation can be converted to the equivalent form

$$(H_1/H_2) = (C_{t2}/C_{t1})^{+4/3}(T_{21}/T_{11})^{-4/3} \tag{35}$$

Making use of equations (32) and (35) and the above stated values of B_1, B_2, L_s/M_s, and L_t/M_t, Eq. (19) simplifies to the CQC form

$$\frac{|v_{\mathrm{R}}(t)|_{\max}}{S_{\mathrm{d}}(\xi, T_{11})} = \frac{3}{2}\left[1 - 2\rho_{\mathrm{e}12}\left(\frac{C_{t2}}{C_{t1}}\right)^{4/3}\left(\frac{T_{21}}{T_{11}}\right)^{-1/3} + \left(\frac{C_{t2}}{C_{t1}}\right)^{8/3}\left(\frac{T_{21}}{T_{11}}\right)^{-2/3}\right]^{1/2} \tag{36}$$

after dividing both sides by $S_{\mathrm{d}}(\xi, T_{11})$. The corresponding SRSS and ABS relations are

$$\frac{|v_{\mathrm{R}}(t)|_{\max}}{S_{\mathrm{d}}(\xi, T_{11})} = \begin{cases} \dfrac{3}{2}\left[1 + \left(\dfrac{C_{t2}}{C_{t1}}\right)^{8/3}\left(\dfrac{T_{21}}{T_{11}}\right)^{-2/3}\right]^{1/2} & (37) \\[2ex] \dfrac{3}{2}\left[1 + \left(\dfrac{C_{t2}}{C_{t1}}\right)^{4/3}\left(\dfrac{T_{21}}{T_{11}}\right)^{-1/3}\right] & (38) \end{cases}$$

respectively.

Equations (36)–(38) apply to that case where the building fundamental period T is proportional to $H^{3/4}$ as indicated in Equation (33), i.e. buildings of the same height and structural type have the same period. Another case of interest is where both buildings have the same height ($H_1 = H_2$) but they have different fundamental periods due to structural differences. As for the previous case, assume that the maximum values of $Y_{11}(t)$ and $Y_{21}(t)$ for the non-linear hysteretic model are approximately the same as for their linear elastic models and that the displacement response spectrum $S_{\mathrm{d}}(\xi, T)$ is proportional to T over the period range $0{\cdot}5 < T < 3{\cdot}0$. Again L_s/M_s and L_t/M_t both equal 3/2 due to the assumed linear mode shape and uniform mass distribution; however, in this case $B_2 = B_1 = 1$. Letting $\xi_1 = \xi_2 = \xi$, Equation (19) now simplifies to the CQC form

$$\frac{|v_{\mathrm{R}}(t)|_{\max}}{S_{\mathrm{d}}(\xi, T_{11})} = \frac{3}{2}\left[1 - 2\rho_{\mathrm{e}12}\left(\frac{T_{21}}{T_{11}}\right) + \left(\frac{T_{21}}{T_{11}}\right)^2\right]^{1/2} \tag{39}$$

if dividing both sides by $S_{\mathrm{d}}(\xi, T_{11})$. The corresponding SRSS and ABS relations are

$$\frac{|v_{\mathrm{R}}(t)|_{\max}}{S_{\mathrm{d}}(\xi, T_{11})} = \begin{cases} \dfrac{3}{2}\left[1 + \left(\dfrac{T_{21}}{T_{11}}\right)^2\right]^{1/2} & (40) \\[2ex] \dfrac{3}{2}\left[1 + \left(\dfrac{T_{21}}{T_{11}}\right)\right] & (41) \end{cases}$$

NUMERICAL EXAMPLE

Case 1: Consider two steel moment-resisting frame buildings subjected to an earthquake excitation corresponding to the 5 per cent-damped S_2 soil-type (deep cohensionless or stiff clay soils) normalized

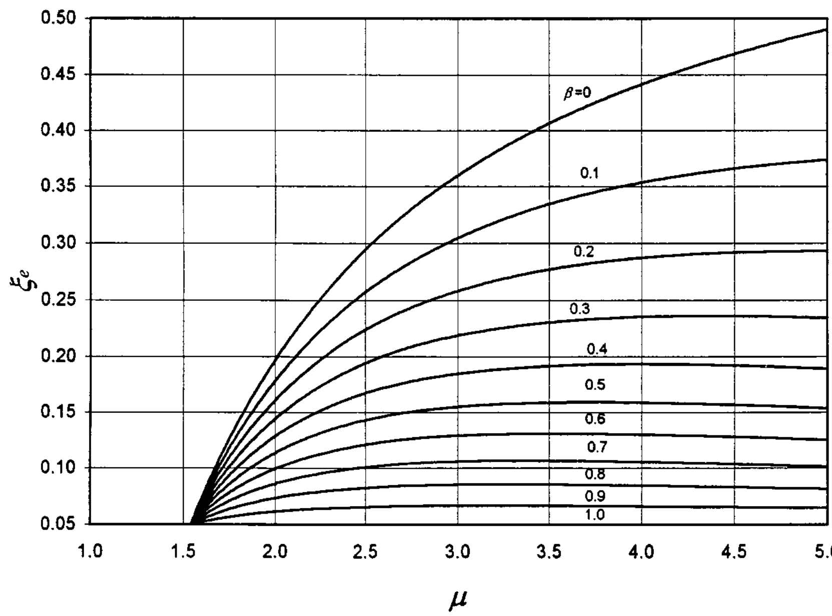

Figure 6. Equivalent linearized damping ratio as a function of maximum ductility demand for discrete values of $\beta(\beta \equiv \bar{K}/K)$

acceleration response spectrum in the 1994 Uniform Building Code[5] (Vol. 2, Fig. 16–3, pp. 2–42) with the effective peak ground acceleration specified to be 0·4 g. Over the period range $0{\cdot}58 < T < 3{\cdot}00$, this normalized spectrum is given by

$$S_a(0{\cdot}05, T)/0{\cdot}4g = 1{\cdot}45/T \tag{42}$$

which corresponds to the displacement response spectrum

$$S_d(0{\cdot}05, T) = 14{\cdot}4T \tag{43}$$

in which units of centimeters and seconds are used.

Assuming $\beta = 0{\cdot}05$ for both buildings and that they have been designed for a maximum ductility demand, μ, equal to 3·0 when subjected to the above specified seismic input, ξ_e equals 0·33 as shown in Figure 6. Because μ and β are the same for both buildings, the period ratio T_{21}/T_{11} representing linear elastic behaviour equals the corresponding period ratio T_{e21}/T_{e11} representing the equivalent linearized behaviour. Using the results in Figure 2 for $\xi = \xi_e = 0{\cdot}33$, values of ρ_{e12} can be obtained for discrete values of T_{21}/T_{11}. Letting the ratio C_{t2}/C_{t1} equal unity, consistent with both buildings being of the same structural type, and introducing the values of ρ_{e12} into equation (36), the desired response ratio $|v_R(t)|_{max}/S_d(\xi, T_{11})$ can be obtained as a function of T_{21}/T_{11}. This function, designated CQC, is shown in Figure 7 along with the corresponding SRSS and ABS functions obtained from equations (37) and (38), respectively. The abscissa scale in this figure is also shown in terms of the building-height ratio H_2/H_1 in conformance with equation (34).

Case 2: Consider two buildings of the same height ($H_1 = H_2$) but having different periods due to structural differences. Subject both buildings to the same seismic excitation as characterized by the displacement response spectrum shown in equation (43). Using the same values of μ and β as in the previous Case 1 which resulted in $\xi_e = 0{\cdot}33$, evaluating ρ_{e12} for discrete values of T_{21}/T_{11}, the response ratio $|v_R(t)|_{max}/S_d(\xi, T_{11})$ expressed by equation (39) can be obtained as a function of T_{21}/T_{11}. This function, designated CQC, is shown in Figure 8 along with the corresponding SRSS and ABS functions obtained from

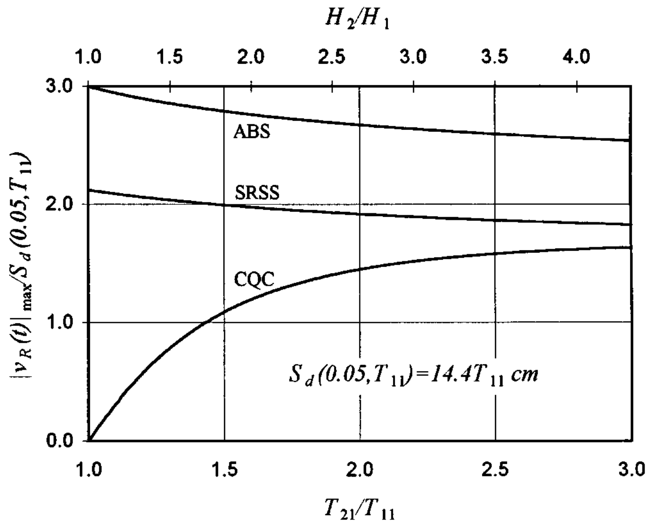

Figure 7. Normalized building separation to prevent pounding: Case 1

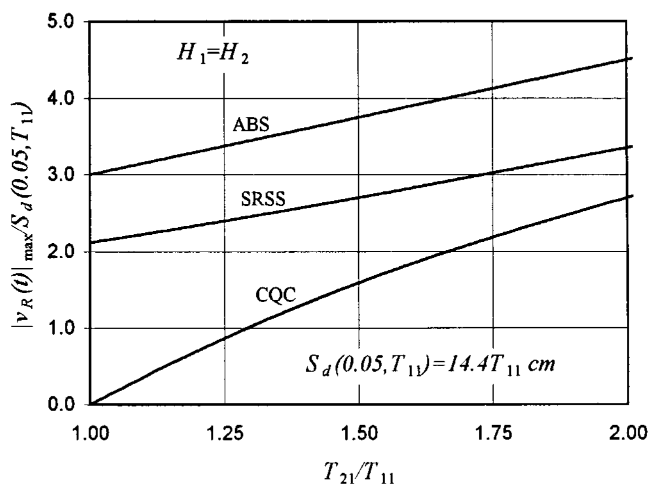

Figure 8. Normalized building separation to prevent pounding: Case 2

equations (40) and (41), respectively. Note the results in this case are significantly different from those in Figure 7 for Case 1.

CONCLUDING REMARKS

One should realize that predicting the fundamental periods of two buildings located adjacent to each other is often subject to considerable error. Therefore, such variations should be taken into account when evaluating separation distance. In addition, one should consider the possibility of the displacement response spectral values being larger than specified in the code. The exceedance probabilities for a given period of time associated with the normalized inward relative displacement values given by the CQC relations in Figures 7

and 8 would be similar to the exceedance probabilities of the response spectral values used. All uncertainties should be considered when evaluating or specifying separation distances.

REFERENCES

1. A. Der Kiureghian, 'Structural response to stationary excitation', *J. eng. mech. div. ASCE*, December 1980.
2. R. W. Clough and J. Penzien, *Dynamics of Structures*, 2nd edn., McGraw-Hill, New York, 1993.
3. V. Jeng, K. Kasai and B. F. Maison, 'A Spectral difference method to estimate building separations to avoid pounding', *Earthquake spectra, J. Earthquake Engineering Research Institute* (*EERI*) **8** (1992).
4. H. B. Seed and I. M. Idriss, 'Soil moduli and damping factors for dynamic response analyses', *Report No. EERC 70-10,* Earthquake Engineering Research Center, University of California, Berkeley, CA, 1970.
5. International Conference of Building Officials, *Structural Engineering Provisions*, Uniform Building Code, Vol. 2, 1994.

